Girraween High School

Year 12 HSC Trial Examination

August 2014

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11–16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Circle the letter corresponding to the correct answer.

Question 1 (1 mark)

What is the correct value of $|-1|-|-1|$?

A. 0

B. -1

C. 1

D. 2

Question 2 (1 mark)

What is 0.01077 rounded to 3 significant figures?

A. 0.0107

B. 0.011

C. 0.0108

D. 0.01

Question 3 (1 mark)

Solve the equation $3x-1=\dfrac{x+3}{2}$

A. $x=\dfrac{4}{5}$

B. $x=1$

C. $x=2$

D. $x=7$

Question 4 (1 mark)

Which is the correct factorisation of $3x^2+x-2$?

A. $(3x+2)(x-1)$

B. $(3x-2)(x+1)$

C. $(3x-1)(x+2)$

D. $(3x+1)(x-2)$

Question 5 (1 mark)

What is the distance between the points $(6, 1)$ and $(3, -3)$?

A. 5

B. 25

C. $\sqrt{7}$

D. 7

Question 6 (1 mark)

When the denominator is rationalised, $\dfrac{1}{\sqrt{3}-\sqrt{2}} =$

A. $\dfrac{\sqrt{3}-\sqrt{2}}{5}$

B. $\sqrt{3}-\sqrt{2}$

C. $\dfrac{\sqrt{3}+\sqrt{2}}{5}$

D. $\sqrt{3}+\sqrt{2}$

Question 7 (1 mark)

What is the domain of the function $f(x) = \sqrt{6-2x}$?

A. All real x such that $x \leq 3$

B. All real x such that $x < 3$

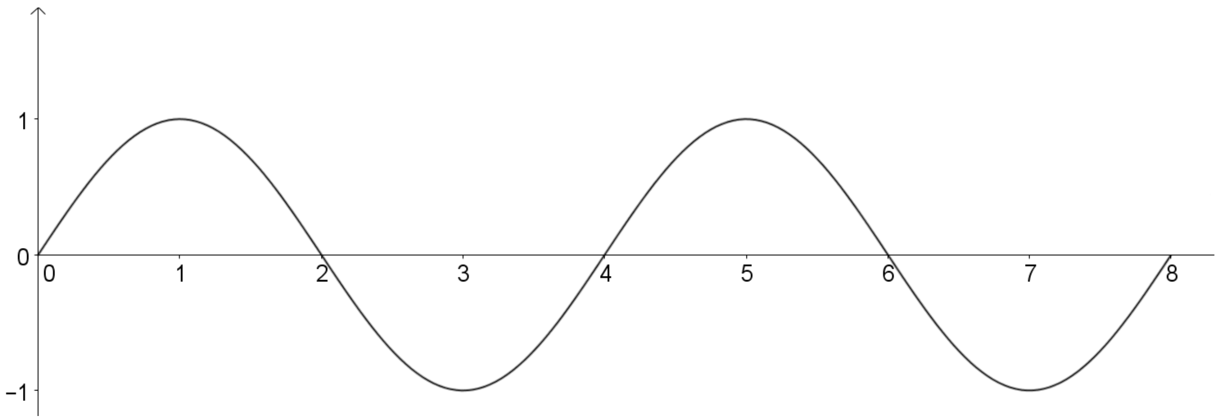
C. All real x such that $x \geq 3$

D. All real x such that $x > 3$

Question 8 on the next page

Question 8 (1 mark)

Below is a graph of $y = \sin \dfrac{\pi x}{2}$. Which of the following integrals has the greatest value?



A. $\displaystyle\int_0^1 \sin \frac{\pi x}{2}\, dx$

B. $\displaystyle\int_0^2 \sin \frac{\pi x}{2}\, dx$

C. $\displaystyle\int_0^4 \sin \frac{\pi x}{2}\, dx$

D. $\displaystyle\int_0^7 \sin \frac{\pi x}{2}\, dx$

Question 9 (1 mark)

When x is replaced with $x + 1$ in the equation $y = x^2$, the graph is moved:

A. One unit to the right

B. One unit to the left

C. One unit higher

D. One unit lower

Question 10 (1 mark)

Which of the following is the correct function value at the minimum turning point of:

$$f(x) = (x - 2012)(x - 2013)(x - 2014)$$

A. -1

B. 0

C. $\dfrac{1}{2}(2012)(2013)(2014)$

D. $\dfrac{-2\sqrt{3}}{9}$

Section II

90 marks

Attempt Questions 11-16

Allow about 2 hours and 45 minutes for this section

Write your answers on the paper provided.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) Evaluate $\sqrt{\pi^2 + 5}$ to two decimal places. [1]

(b) Convert $\frac{3\pi}{5}$ radians to degrees. [1]

(c) Find the exact value of $\sin \frac{2\pi}{3}$ [1]

(d) Simplify $\frac{x}{x^2 - 4} + \frac{2}{x - 2}$ [2]

(e) Find the values of x for which $|x - 3| \geq 3$ [2]

(f) Differentiate with respect to x

i. $y = x^2 \ln 3x$ [2]

ii. $y = \frac{\sin 4x}{x^3}$ [2]

(g) Find $\int 1 + \sec^2 x \; dx$ [2]

(h) Find the limiting sum of the geometric series [2]

$$\frac{3}{4} + \frac{3}{16} + \frac{3}{64} + ...$$

The exam continues on the next page

Question 12 (15 marks)

(a) A function $f(x)$ passes through the point $(2, 10)$. Given that [3]

$$f'(x) = 3x^2 - 3x + 5$$

find the value of $f(1)$.

(b) In a certain arithmetic series, the first term is 13 and the sixth term is -7.

i. Find the common difference. [1]

ii. Find the value of the third term. [2]

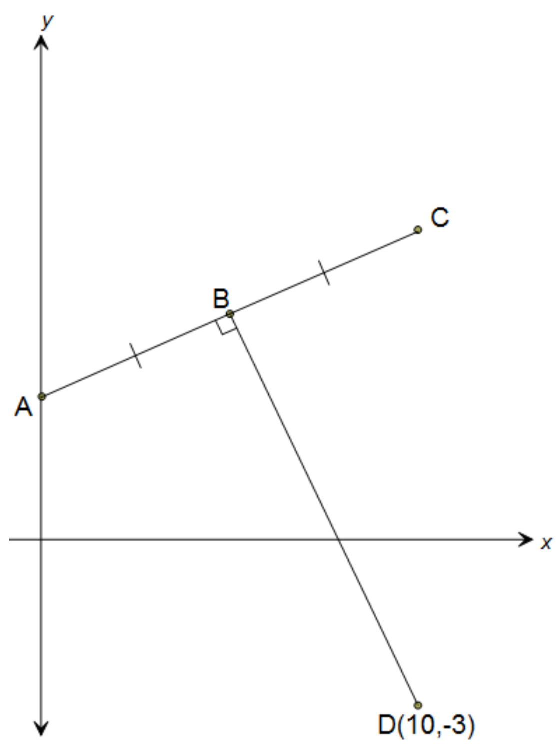
(c) Consider the parabola $y + 2 = 8(x - 1)^2$. Find:

i. the coordinates of the vertex, [1]

ii. the coordinates of the focus, [2]

iii. the equation of the directrix. [1]

(d) The diagram shows points A, B and C lying on the line $2y = x + 4$. The point A lies on the y-axis and $AB = BC$. The line from $D(10, -3)$ to B is perpendicular to AC.



i. Find the coordinates of A. [1]

ii. Find the equation of the line BD. [2]

iii. Find the coordinates of C. [2]

The exam continues on the next page

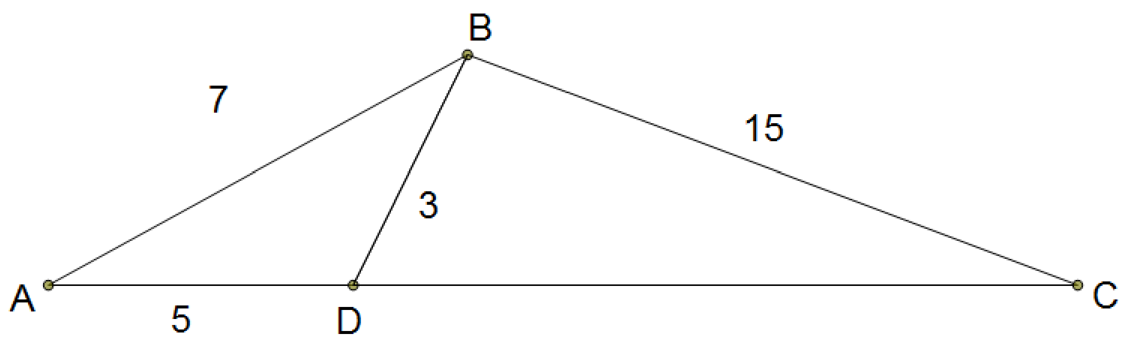
Question 13 (15 marks)

(a) Let α and β be the roots of $2x^2 - 4x - 2 = 0$.

i. State the value of $\alpha\beta$ [1]

ii. Find $\dfrac{5}{\alpha} + \dfrac{5}{\beta}$ [2]

(b) In the diagram below, triangle ABC has dimensions $AB = 7cm$ and $BC = 15cm$. The point D lies on AC such that $AD = 5cm$ and $BD = 3cm$.

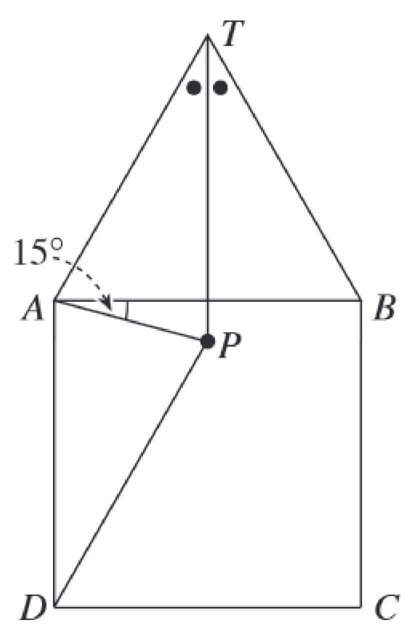


i. Use the cosine rule to show that $\angle ADB = 120°$. [2]

ii. Show that $\angle BCD = 10°$ (rounded to the nearest degree). [2]

iii. Find the length of DC, correct to the nearest cm. [2]

(c) In the diagram below, $ABCD$ is a square and ABT us an equilateral triangle. The line TP bisects $\angle ATB$, and $\angle PAB = 15°$.



i. Copy the diagram into your writing booklet and explain why $\angle PAT = 75°$. [1]

ii. Prove that $\Delta TAP \equiv \Delta DAP$. [3]

iii. Prove that ΔDAP is isosceles. [2]

The exam continues on the next page

Question 14 (15 marks)

(a) Consider the function $f(x) = x^4 - 4x^3$.

i. Show that $f'(x) = 4x^2(x-3)$ [1]

ii. Find the coordinates of the stationary points of the curve $f(x)$, and determine their nature. [3]

iii. Sketch the graph of the curve $f(x)$, showing the stationary points. [1]

iv. Find the values of x for which the graph of $f(x)$ is concave down. [2]

(b) Jenny borrows \$500 000 to buy a house. An interest rate of 9% p.a. compounded monthly is charged on the outstanding balance. The loan is to be repaid in equal monthly instalments of $\$M$ over a 25 year period (300 months). Let A_n be the amount owing after n months.

i. Show the amount owing after 3 months is: [2]

$$A_3 = 500000 \times 1.0075^3 - M(1 + 1.0075 + 1.0075^2)$$

ii. Find the required monthly repayment. [2]

iii. How much interest does Jenny pay over the 25 years? [1]

(c) A bag contains six discs. Two of the discs have the number 0 on them and the other four discs have the number 1 on them. Three discs are drawn at random without replacement.

i. Find the probability that all of the three discs drawn have the number 1 on them. [2]

ii. Find the probability that the product of the numbers on the three discs drawn is 0. [1]

The exam continues on the next page

Question 15 (15 marks)

(a) Suppose $y = \sqrt{3^x + x}$:

i. Complete the table below, giving the values of y to 3 decimal places. [2]

x	0	0.25	0.5	0.75	1
y	1	1.251			2

ii. Use the trapezoidal rule with all the values of y from the table above to find an approximation for the value of [3]

$$\int_0^1 \sqrt{3^x + x}\, dx$$

(b) The mass (kg) of a decaying substance at time t years is given by $M = M_0 e^{-kt}$, where M_0 is its initial mass and k is a positive constant.

i. Show that M satisfies the differential equation $\frac{dM}{dt} = -kM$. [1]

ii. Show that the half-life of the substance is given by $\frac{\ln 2}{k}$ years. [2]

iii. A second substance is also decaying simultaneously with the first substance but its rate of decay is twice as fast. Find the mass of the second radioactive substance N in terms of M_0 and k, given that its initial mass is half of the first substance. [3]

(c) The velocity of a particle moving along the x axis at time t is given by

$$v = te^{2t}$$

i. Find the acceleration of the particle. [1]

ii. Given that when $t = 0$ the particle is at $x = 0$, find x in terms of t. [3]

The exam continues on the next page

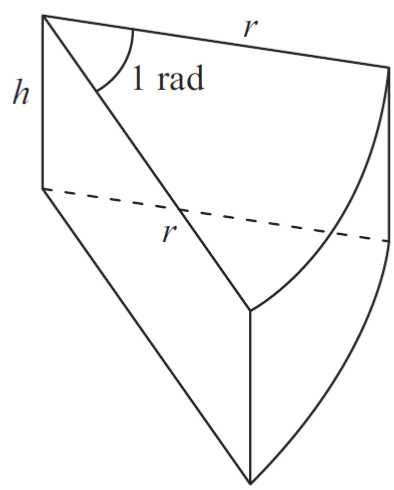
Question 16 (15 marks)

(a) i. Given $2\log_3 x - \log_3(x-2) = 2$ show that x satisfies $x^2 - 9x + 18 = 0$. [3]

ii. Hence, or otherwise, solve the equation: [1]

$$2\log_3 x - \log_3(x-2) = 2$$

(b) The diagram below shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian. The volume of the box is 300 cm^3.



i. Show that the surface area of the box, S cm^2, is given by: [3]

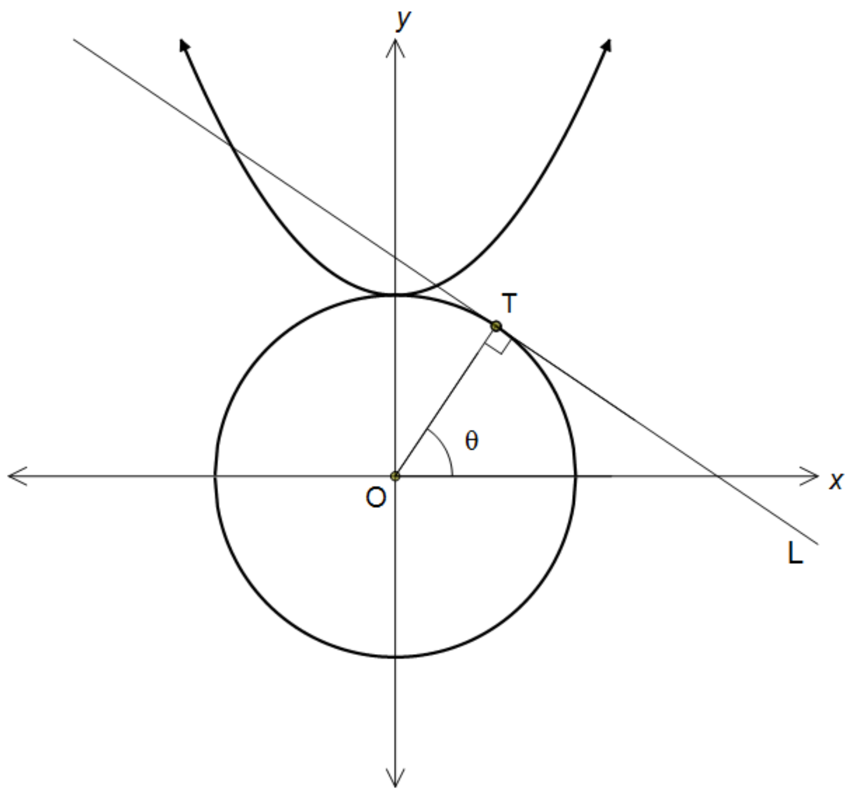
$$S = r^2 + \frac{1800}{r}$$

ii. Find the value of r that minimises the the value of S. Give your answer correct to one decimal place. [3]

The exam continues on the next page

(c) The diagram below shows the graph of $y = x^2 + 1$ and a point T on the unit circle $x^2 + y^2 = 1$ at angle θ from the positive x-axis, where $0 \leq \theta \leq 2\pi$.

The tangent line L to the circle at T is perpendicular to OT.



i. Show that the equation of the tangent line L is given by [2]

$$x \cos\theta + y \sin\theta = 1$$

ii. Show that if L intersects $y = x^2 + 1$ twice then $\sin\theta$ satisfies the following inequalities [2]

$$-\frac{1}{5} < \sin\theta < 0 \text{ or } 0 < \sin\theta < 1$$

iii. Find the values of θ to the nearest minute such that L intersects $y = x^2 + 1$ twice. [1]

End of exam

Year 12 HSC TRIAL 2014

Multiple Choice Solutions:

1. A 2. C 3. B 4. B 5. A

6. D 7. A 8. B 9. B 10. D

Q1

$|-1| - |-1| = 1 - 1 = 0 \quad \therefore \text{(A)}$

Q2

$= 0.0108 \quad \therefore \text{(C)}$

Q3

$3x - 1 = \dfrac{x+3}{2}$

$6x - 2 = x + 3$

$6x - x = 2 + 3$

$5x = 5$

$x = 1 \quad \therefore \text{(B)}$

Q4

$3x^2 + x - 2$

$3x \quad -2$

$x \quad 1$

$(3x-2)(x+1) \quad \therefore \text{(B)}$

Q5

$D = \sqrt{(6-3)^2 + (1+3)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

$\therefore \text{(A)}$

Q6

$\dfrac{1}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \dfrac{\sqrt{3}+\sqrt{2}}{3-2} = \sqrt{3}+\sqrt{2} \quad \therefore \text{(D)}$

Q7

$6 - 2x \geq 0$

$6 \geq 2x$

$2x \leq 6$

$x \leq 3 \quad \therefore \text{(A)}$

Q8

$\int_0^4 y\,dx = 0$

$0 < \int_0^1 y\,dx = \int_0^7 y\,dx$

$\int_0^2 y\,dx = 2\int_0^1 y\,dx$

$\therefore \text{(B)}$

Q9

x replaced with $x+1$ moves the graph 1 unit to the left.

$\therefore \text{(B)}$

Q10

The value and the nature of turning points does not change under translations of the graph.

So replacing x with $x+2013$ in $f(x)$ we get:

$f(x) = (x+1)x(x-1)$

$f(x) = x(x+1)(x-1)$

$f(x) = x(x^2-1)$

$f(x) = x^3 - x$

$f'(x) = 3x^2 - 1$

$f'(x) = 0$ when $3x^2-1=0$

$x^2 = \frac{1}{3}$ $\quad x = \pm\frac{1}{\sqrt{3}}$

$f''(x) = 6x$ so $f''\left(\frac{1}{\sqrt{3}}\right) = 6\times\frac{1}{\sqrt{3}} > 0$

so $x = \frac{1}{\sqrt{3}}$ gives the minimum

$f\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}\left(\frac{1}{3}-1\right) = \frac{1}{\sqrt{3}}\times -\frac{2}{3}$

$= \frac{-2}{3\sqrt{3}} = \frac{-2\sqrt{3}}{9}$

$\therefore$ (D)

Q11

(a) 3.86 (2 dp).

(b) $\frac{3\pi}{5} = \frac{3\pi}{5}\times\frac{180}{\pi} = 108°$

(c) $\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

(d) $\frac{x}{x^2-4} + \frac{2}{x-2}$

$= \frac{x}{x^2-4} + \frac{2(x+2)}{x^2-4}$

$= \frac{x+2x+4}{x^2-4} = \frac{3x+4}{x^2-4}$

(e) $|x-3| \geq 3$

$x-3 \geq 3$ & $3-x \geq 3$

$x \geq 6$ & $3-3 \geq x$

$x \leq 0$

(f)

(i) $y = x^2 \ln 3x$

$u = x^2$ $\quad v = \ln 3x$

$u' = 2x$ $\quad v' = \frac{1}{x}$

$y' = 2x\ln 3x + x.$

Q11

(f)

(ii) $y = \frac{\sin 4x}{x^3}$

$u = \sin 4x \qquad v = x^3$

$u' = 4\cos 4x \qquad v' = 3x^2$

$y' = \frac{4x^3\cos 4x - 3x^2 \sin 4x}{x^6}$

$y' = \frac{4x\cos 4x - 3\sin 4x}{x^4}$

(g) $\int 1 + \sec^2 x \, dx$

$= x + \tan x + c$

(h) $a = \frac{3}{4} \quad r = \frac{1}{4}$

$S_\infty = \frac{a}{1-r} = \frac{\frac{3}{4}}{1-\frac{1}{4}} = \frac{\frac{3}{4}}{\frac{3}{4}} = 1$

Q12

(a) $f'(x) = 3x^2 - 3x + 5$

$f(x) = x^3 - \frac{3}{2}x^2 + 5x + C$

$f(2) = 10$

So $10 = 2^3 - \frac{3}{2} \times 2^2 + 5 \times 2 + C$

$10 = 8 - 6 + 10 + C$

$C = 6 - 8 = -2$

$\therefore f(x) = x^3 - \frac{3}{2}x^2 + 5x - 2$

$\therefore f(1) = 1 - \frac{3}{2} + 5 - 2$

$f(1) = \frac{5}{2}$.

(b)

i. $a = 13$

$T_6 = a + (6-1)d$

$-7 = 13 + 5d$

$5d = -20$, so $d = -4$

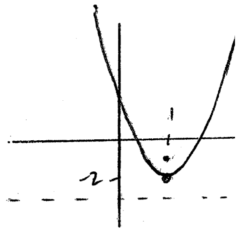
ii. $T_3 = 13 + 2 \times -4$

$= 13 - 8 = 5$

(c)

(i) $\frac{1}{8}(y+2) = (x-1)^2$

So $V = (1, -2)$



(ii) $4a = \frac{1}{8}$ so $a = \frac{1}{32}$

$\therefore F = (1, -2 + \frac{1}{32}) = (1, -\frac{63}{32})$

(iii) D: $y = -2\frac{1}{32}$

(d) (i) $y = \frac{x}{2} + 2$

So $A = (0, 2)$

(ii) m of $BD = -2$ as $-2 \times \frac{1}{2} = -1$

So $m = -2$ & point $= (10, -3)$

$y + 3 = -2(x - 10)$

$y = -2x + 20 - 3$

$y = -2x + 17$

Q12

(d)

(iii) Finding intersection of line AC & BD:

$2y = x+4$

$y = -2x+17$

So $2(-2x+17) = x+4$

$-4x + 34 = x+4$

$30 = 5x$

$x = 6$

$\therefore y = -2\times 6 + 17 = 5$.

$\therefore B = (6, 5)$

Since B is the midpoint of AC

$6 = \frac{x+0}{2}$ & $5 = \frac{2+y}{2}$

$\therefore x = 12$ $2+y = 10$

$y = 8$

$\therefore C = (12, 8)$

Q13

(a)

(i) $\alpha\beta = \frac{c}{a} = \frac{-2}{2} = -1$.

(ii) $\frac{5}{\alpha} + \frac{5}{\beta} = \frac{5\beta + 5\alpha}{\alpha\beta}$

$= \frac{5(\alpha+\beta)}{\alpha\beta}$

$= \frac{5 \times \frac{-(-4)}{2}}{-1}$

$= \frac{10}{-1} = -10$.

(b)

(i) $\cos\angle ADB = \frac{5^2+3^2-7^2}{2\times 5\times 3}$

$\cos\angle ADB = -\frac{1}{2}$

$\therefore \angle ADB = 180 - 60 = 120^\circ$

(ii) $\angle BDC = 60$ ($\angle$ in a straight line)

Using sine rule in $\triangle BDC$:

$\frac{\sin\angle BCD}{3} = \frac{\sin 60}{15}$

$\sin\angle BCD = \frac{\sqrt{3}}{2} \times \frac{1}{5} = \frac{\sqrt{3}}{10}$.

$\angle BCD = \sin^{-1}\left(\frac{\sqrt{3}}{10}\right) = 10^\circ$ (nearest deg.)

(iii) $\frac{DC}{\sin\angle DBC} = \frac{15}{\sin 60}$

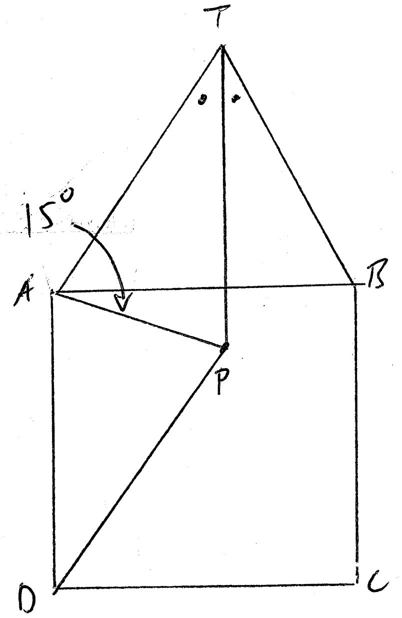
$\angle DBC = 180 - 60 - \angle BCD$ ($\angle$ sum of a $\triangle$)

$DC = \frac{15\sin\angle DBC}{\sin 60} = 16$ cm (nearest cm)

Q13

(c)

(i)



$\angle TAB = 60°$ ($\angle$ in an equilateral Δ)

$\therefore \angle PAT = 60 + 15 = 75°$

(ii) $AD = AB$ (Sides of a square)

$AB = AT$ (Sides of an equilateral Δ)

$\therefore AD = AT$

$\angle DAP = 90 - 15 = 75°$ ($\angle$ in a square)

$\therefore \angle DAP = \angle PAT = 75°$

AP is common.

$\therefore \Delta TAP \equiv \Delta DAP$ (SAS)

(iii) $\angle ATP = 30°$ (TP bisects $\angle ATB$)

$\therefore \angle TPA = 180 - 30 - 75 = 75°$ ($\angle$ sum of Δ)

$\therefore AT = TP$ (Equal sides opposite equal $\angle$'s in Isosceles ΔATP)

But since $\Delta APD \equiv \Delta ATP$, ΔAPD must also be Isosceles.

Q14

(a)

(i) $f(x) = x^4 - 4x^3 = x^3(x-4)$

$f'(x) = 4x^3 - 12x^2$

$= 4x^2(x-3)$

(ii) $f'(x) = 0$ when $4x^2(x-3) = 0$

So $x = 0$ or $x = 3$

$f(0) = 0$ & $f(3) = -27$

So stat pts are $(0,0)$ & $(3,-27)$

$f''(x) = 12x^2 - 24x$

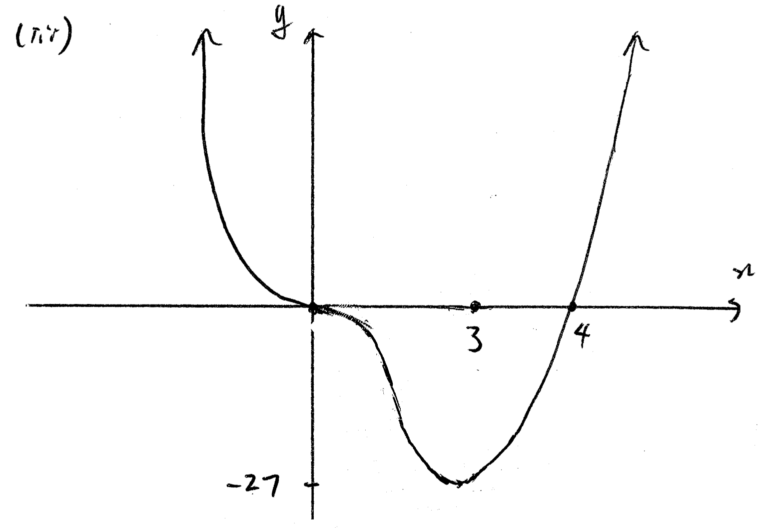
$f''(x) = 12x(x-2)$

x	-1	0	1
$f'(x)$	-16	0	-8

$\therefore (0,0)$ is a horizontal point of inflexion.

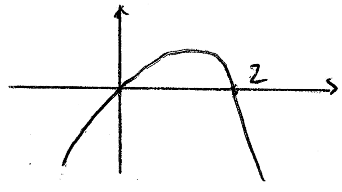
$f''(3) = 36 > 0 \therefore (3,-27)$ a local minimum.

(iii)



(iv) Concave down when $f''(x) < 0$

So $12x(x-2) < 0$



So $0 < x < 2$

Q14

(b)

(i) $R = \frac{9}{12} \div 100 = 0.0075$.

$A_1 = 500000(1.0075) - M$

$A_2 = A_1(1.0075) - M$

$A_2 = 500000(1.0075)^2 - M(1.0075) - …$

$A_3 = A_2(1.0075) - M$

$= 500000(1.0075)^3 - M(1.0075)^2 - M(1.0075) - M$

$= 500000(1.0075)^3 - M(1 + 1.0075 + 1.007…$

(ii)

$A_n = 500000(1.0075)^n - M(1 + 1.0075 + 1.0075 + \ldots + 1.0075^{n-1})$

$A_{300} = 500000(1.0075)^{300} - M\left(\frac{1.0075^{300} - 1}{1.0075 - 1}\right)$

$A_{300} = 0$ when:

$M = 500000(1.0075)^{300} \times \frac{0.0075}{1.0075^{300} - 1}$

$M = \$4195.98$

(iii) $I = 4195.98 \times 300 - 500000$

$= \$758794.55$

Q14

(c)

(i) $p = \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} = \frac{1}{5}$

(ii) p(atleast one 0) = 1 − p(all 1's)

$= 1 - \frac{1}{5} = \frac{4}{5}.$

Q15

(a)

(i)

x	0	0.25	0.5	0.75	1
y	1	1.251	1.494	1.741	2

(ii) $h = 0.25$

$A \approx \frac{0.25}{2}\left[1 + 2\times 1.251 + 2\times 1.494 + 2\times 1.741 + 2\right]$

$A \approx 1.4965$

(b)

(i) $M = M_0 e^{-kt}$

$\frac{dM}{dt} = -k M_0 e^{-kt}$

$= -kM$

(ii) $\frac{1}{2} M_0 = M_0 e^{-kt}$

$\frac{1}{2} = e^{-kt}$

$-kt = \ln \frac{1}{2}$

$-kt = \ln 1 - \ln 2$

$kt = \ln 2 \quad \therefore \quad t = \frac{\ln 2}{k}$

(iii) $\frac{dN}{dt} = -2k M_0 e^{-kt}$

$\therefore N = -2kM_0 \int e^{-kt}\, dt$

$N = 2M_0 e^{-kt} + C$

When $t = 0$ $N = \frac{1}{2} M_0$

So $\frac{1}{2} M_0 = 2M_0 + C$

$\therefore C = -\frac{3}{2} M_0$

$\therefore N = 2M_0 e^{-kt} - \frac{3}{2} M_0$

(c)

(i) $v = t e^{2t}$

$u = t \quad v = e^{2t}$

$u' = 1 \quad v' = 2e^{2t}$

$a = e^{2t} + 2te^{2t}.$

(ii) $\int a\, dt = v$

So $\int e^{2t} + 2te^{2t}\, dt = te^{2t}$

So $\int e^{2t}\, dt + 2\int te^{2t}\, dt = te^{2t}$

So $\frac{1}{2} e^{2t} + 2\int v\, dt = te^{2t}$

Q15

(c)

(ii) $2\int v\,dt = te^{2t} - \frac{1}{2}e^{2t}$

$2x = te^{2t} - \frac{1}{2}e^{2t}$

$x = \frac{1}{2}te^{2t} - \frac{1}{4}e^{2t} + c$

$t=0 \quad x=0$

$0 = 0 - \frac{1}{4} + c \quad \therefore c = \frac{1}{4}.$

$\therefore x = \frac{1}{2}te^{2t} - \frac{1}{4}e^{2t} + \frac{1}{4}.$

Q16

(a) $2\log_3 x - \log_3(x-2) = 2.$

(i) $\log_3(x^2) - \log_3(x-2) = 2$

$\log_3\left(\frac{x^2}{x-2}\right) = 2$

$\frac{x^2}{x-2} = 3^2$

$x^2 = 9(x-2)$

$x^2 = 9x - 18$

$x^2 - 9x + 18 = 0.$

(ii) $x^2 - 9x + 18 = 0$

$x \quad -3$

$x \quad -6$

$(x-3)(x-6) = 0$

$x = 3, \ x = 6.$

(b)

(i)

Area of cross section $= \frac{1}{2} \times 1 \times r^2$

$= \frac{1}{2}r^2$

$\therefore V = \frac{1}{2}hr^2$

$\therefore 300 = \frac{1}{2}hr^2$

$\therefore h = \frac{600}{r^2}$

$S = 2\left(\frac{1}{2}r^2\right) + 2(hr) + (rh)$

$S = r^2 + 2hr + rh$

$S = r^2 + 3hr$

$S = r^2 + \frac{1800}{r}$

(ii) $S = r^2 + 1800r^{-1}$

$S' = 2r - 1800r^{-2}$

$S' = 2r - \frac{1800}{r^2}$

$S' = 0$ when $2r = \frac{1800}{r^2}$

$2r^3 = 1800$

$r^3 = 900$

$r = \sqrt[3]{900}$

$S'' = 2 + 3600r^{-3} = 2 + \frac{3600}{r^3}$

$S'' = 2 + \frac{3600}{900} > 0 \quad \therefore r = \sqrt[3]{900}$ gives the minimum

$r = 9.7\text{cm (1dp)}$

Q16

(c)

(i) $T = (\cos\theta, \sin\theta)$

$M_{OT} = \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\therefore$ m of $L = -\frac{\cos\theta}{\sin\theta}$

$\therefore$ Equation of L is given by

$y - \sin\theta = -\frac{\cos\theta}{\sin\theta}(x - \cos\theta)$

$y\sin\theta - \sin^2\theta = -\cos\theta(x - \cos\theta)$

$y\sin\theta - \sin^2\theta = -x\cos\theta + \cos^2\theta$

$y\sin\theta + x\cos\theta = \sin^2\theta + \cos^2\theta$

$x\cos\theta + y\sin\theta = 1$

when $\theta = 0$, $x + y\sin 0 = 1$

$x = 1$

when $\theta = 180$, $x\cos 180 + y\sin 180 = 1$

$-x + 0 = 1$

$x = -1$

$\therefore$ L is given by $x\cos\theta + y\sin\theta = 1$.

(ii)

$x\cos\theta + y\sin\theta = 1$

$y = x^2 + 1$

For intersections:

$x\cos\theta + (x^2+1)\sin\theta = 1.$

$(\cos\theta)x + (\sin\theta)x^2 + \sin\theta - 1 = 0.$

$(\sin\theta)x^2 + (\cos\theta)x + (\sin\theta - 1) = 0.$

$x = \frac{-\cos\theta \pm \sqrt{\cos^2\theta - 4(\sin\theta)(\sin\theta - 1)}}{2\sin\theta}$

for $\sin\theta \neq 0$.

So $\Delta = \cos^2\theta - 4(\sin\theta)(\sin\theta - 1)$

$= 1 - \sin^2\theta - 4\sin^2\theta + 4\sin\theta$

$= -5\sin^2\theta + 4\sin\theta + 1$

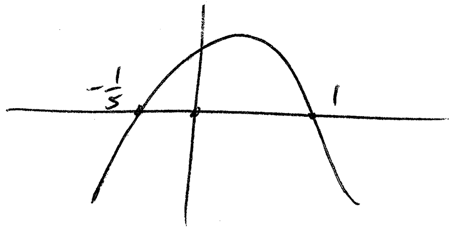
So 2 intersections when

$-5\sin^2\theta + 4\sin\theta + 1 > 0.$

$-5\sin\theta \quad -1$

$\sin\theta \quad -1$

$(-5\sin\theta - 1)(\sin\theta - 1) > 0$



So $-\frac{1}{5} < \sin\theta < 1$ but $\sin\theta \neq 0$

So $-\frac{1}{5} < \sin\theta < 0$ & $0 < \sin\theta < 1$.

Q16/

(c)

(iii) Solving $\sin\theta = -\frac{1}{5}$

$\theta = 180 + \sin^{-1}\left(\frac{1}{5}\right)$ and

$\theta = 360 - \sin^{-1}\left(\frac{1}{5}\right)$

$\theta = 191°32'$ and $348°27'$

$\theta = 191°32'$ and $-11°32'$

$\therefore\ -11°32' < \theta < 191°32'$

and $\theta \neq 0°$ and $\theta \neq 180°$